Thus $g^1{}_{0nl}$ is determined by

$$g^1{}_{0nl}(z)=\frac{n(n-l-1)}{2(n-1)}\int_{-1}^{1}dz'[R(z,z')]^{n-1}(1-z'^2)^n+\frac{n(n+1)}{2(n-1)}\int_{-1}^{1}dz'[R(z,z')]^{n-1}g^1{}_{0nl}(z'). \tag{B3}$$

The homogeneous part of (B3) is satisfied by $g^0{}_{1,n-1,l}$, but it is an odd function of z so that it is quite harmless. By making an ansatz

$$g^1{}_{0nl}(z)=c_0(1-z^2)^n+c_1(1-z^2)^{n-1}, \tag{B4}$$

we can easily obtain

$$g^1{}_{0nl}(z)=\frac{n(n-l-1)}{2(n+1)}[(1-z^2)^n-2(1-z^2)^{n-1}]. \tag{B5}$$

As for $g^2{}_{0nl}$, the homogeneous part of the integral equation for it is satisfied by $g^0{}_{2,n-2,l}$, which is an *even* function of z. Hence, we can no longer find a solution of the type

$$c_0(1-z^2)^n+c_1(1-z^2)^{n-1}+c_2(1-z^2)^{n-2} \tag{B6}$$

as is easily checked.

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Unitary Symmetry and Hyperon Leptonic Decays*

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The consequences (beyond the $\Delta I=\frac{1}{2}$ and $\Delta I=1$ rules) of the most general form of the hypothesis that the weak-interaction currents transform like components of SU_3 octets are discussed. These are presented as relations between the $\Delta S=1$ leptonic decays of Ξ^- and Ξ^0 and those of Σ and Λ. The only prediction for the branching ratios B of Ξ decays which can be compared to present experiments is

$$B(\Xi^-\to\Lambda+e^-+\nu)+B(\Xi^-\to\Sigma^0+e^-+\nu)\leq(1.05\pm0.2)\times10^{-3}.$$

As yet the comparison is inconclusive. One additional relation among the leptonic decays of hyperons is found if the particular model of Cabibbo is assumed. Applications of these considerations to the determination of induced couplings are made.

1. INTRODUCTION

THE success of the unitary symmetry model for strong interactions has led many authors to suggest possible properties of the weak-interaction currents with respect to the SU_3 transformations.[1–4] Practically all the proposals include the hypothesis that each of the nonleptonic weak currents which are coupled to leptons (or to intermediate bosons) transform like components of some octet. The main purpose of the present note is to discuss those experimentally observable consequences that follow from this hypothesis alone and so are common to all the proposals. The present discussion is limited to leptonic decays of hyperons, which are particularly suitable for testing this hypothesis. There exist sixteen possible leptonic decay amplitudes, of which twelve should be observable in the absence of selection rules; the other four either compete with the electromagnetic decay of the Σ^0 or are intrinsically very rare because of their small energy release.

We first review in Sec. 2 those selection rules that may follow from postulating transformation properties of the weak currents with respect to strangeness, isotopic spin, and G. These selection rules, which are well known but not well verified experimentally, provide eight relationships among the sixteen amplitudes. In Sec. 3 we discuss the consequences of the most general form of the octet hypothesis, which provides four additional relationships. The further hypotheses that can be made in an invariant way assuming perfect SU_3 symmetry are discussed in Sec. 5; they lead essentially to the model of Cabibbo, which provides one additional relationship when only hyperon leptonic decays are considered.

The weak interaction responsible for leptonic decays

* This work supported in part by U. S. Atomic Energy Commission.
[1] M. Gell-Mann, California Institute of Technology Report CTSL-20, 1961 (unpublished).
[2] N. Cabibbo and R. Gatto, Nuovo Cimento 21, 872 (1961).
[3] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963); see also B. d'Espagnat and J. Prentki, Nuovo Cimento 24, 497 (1962).
[4] John M. Cornwall and V. Singh, Phys. Rev. Letters 10, 551 (1963).

is assumed to be of the form

$$(G/\sqrt{2})J^{\mu}L_{\mu}^{\dagger}+\text{H.c.}, \tag{1}$$

where L_μ is the usual lepton current. Each of the decays $B\rightarrow b+l+\nu$ is then described by an amplitude $A(B\rightarrow b)$, which is proportional to $\langle b|J^{\mu}|B\rangle$ or to $\langle b|J^{\mu\dagger}|B\rangle$. This amplitude in general may be expressed in terms of six form factors[5,6]

$$\langle b|J^{\mu}|B\rangle=\left(\frac{m_b m_B}{E_b E_B}\right)^{1/2}\bar{u}_b\left(f_1\gamma^{\mu}+\frac{f_2}{m_B}\sigma^{\mu\nu}k_{\nu}+\frac{f_3}{m_B}k^{\mu}\right.$$
$$\left.+g_1\gamma^{\mu}\gamma_5+\frac{g_2}{m_B}\sigma^{\mu\nu}\gamma_5 k_{\nu}+\frac{g_3}{m_B}\gamma_5 k^{\mu}\right)u_B. \tag{2}$$

Each of the form factors $f_1\cdots g_3$ is a function of k^2, where $k^{\mu}=p_B{}^{\mu}-p_b{}^{\mu}$ is the four-momentum transfer, and of the baryon pair (b,B). We shall discuss equations relating different amplitudes, which will be understood to be evaluated at the same value of k^2; such equations are clearly equivalent to the same equations for each of the six form factors. The form factors f_2, f_3, g_2, and g_3 are often referred to as "induced couplings" in the belief that the derivative couplings that would be required in the weak Hamiltonian in order to produce these terms in simple perturbation theory are actually absent, but that strong-interaction effects cause them to appear in matrix elements between physical baryon states. We shall also refer to the form factors f_3 and g_2 as "second-class form factors" in analogy with the terminology of Weinberg,[7] who shows that these vanish for $\langle p|J^{\mu}|n\rangle$ if the standard G condition (see Sec. 2) is assumed. Generalizations of this result are discussed in Sec. 4.

Most of the experimental results available for comparison involve β decay (rather than the μ-decay mode) so that the form factors f_3 and g_3 make negligible contributions. The total β-decay rate W can be expressed in terms of only three form factors if terms in R^2 are neglected, where Rm_B is the maximum electron energy; if we also neglect the variation of the form factors with k^2, then[6]

$$W(B\rightarrow b+e+\nu)=(G^2 m_B{}^5/60\pi^3)R^5(1+R)\times\{f_1{}^2+3g_1{}^2-4Rg_1g_2\}. \tag{3}$$

As discussed in Sec. 4 for the theories explored here, g_2 is expected to be small; if we neglect its contribution we may rewrite Eq. (3) as

$$W(B\rightarrow b+e+\nu)=(\text{UFI value})\times[(g_V{}^2+3g_A{}^2)/4G^2], \tag{4a}$$

where (UFI value) stands for the decay rate predicted by the simple universal $V-A$ interaction,[8] $g_V=Gf_1$, and $g_A=Gg_1$. For consideration of total decay rates in this approximation we may consider each of the amplitudes $A(B\rightarrow b)$ as a vector in a two-dimensional space with x component g_V and y component $\sqrt{3}g_A$ so that the length of the vector gives the decay rate and the amplitudes may be added like vectors (or, equivalently, like complex numbers). Specifically, for all decays

$$|A(B\rightarrow b)|^2=K[W(B\rightarrow b+e+\nu)/(\text{UFI value})], \tag{4b}$$

where K is a constant independent of decay mode.

2. STRANGENESS, ISOTOPIC-SPIN, AND G SELECTION RULES

Given conservation of isotopic spin I and strangeness S the most restrictive condition on J^{μ} that may be compatible with experimental facts is that it is a sum of a $\Delta S=0$, $\Delta\mathbf{I}=1$ part, $\mathcal{J}^{\mu}$, which has the standard behavior under G, and as a $\Delta S=1$, $\Delta\mathbf{I}=\frac{1}{2}$ part, $\mathcal{S}^{\mu}$. By the "standard behavior under G" we mean[7]

$$G\mathcal{J}^{\mu}G^{-1}=-\xi\mathcal{J}^{\mu}, \tag{5}$$

where $\xi=-1$ for vector couplings and $\xi=+1$ for axial-vector couplings. When coupled with time reversal this corresponds to the $\Delta\mathbf{I}=1$ rule of Lee and Yang[9]; however, we use the notation $\Delta\mathbf{I}=1$ to signify a different property, namely, that all matrix elements $\langle c|\mathcal{J}^{\mu}|0\rangle$ vanish unless $\langle c|$ has $I=1$ with $I_z=1$. These restrictions provide eight conditions on the sixteen possible amplitudes for hyperon leptonic decay. It should be emphasized that there is very little empirical evidence supporting these selection rules.

Two of the conditions correspond to the $\Delta Q=\Delta S$ selection rule:

$$A(\Sigma^+\rightarrow n)=0, \tag{6a}$$

$$A(\Xi^0\rightarrow\Sigma^-)=0. \tag{6b}$$

Considerable evidence now exists[10] for Eq. (6a); the $\Delta Q=\Delta S$ rule is also supported by the apparent forbiddenness[11] of $K^+\rightarrow\pi^++\pi^++e^-+\nu$. Two other conditions yield the forbiddenness of $\Delta S=2$ leptonic decays:

$$A(\Xi^-\rightarrow n)=0, \tag{6c}$$

$$A(\Xi^0\rightarrow p)=0. \tag{6d}$$

Some evidence for this selection rule exists and hopefully more should soon be available. The forbiddenness of $\Delta S=2$ and also $\Delta S=3$ currents can be tested if the

[5] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 35 (1958).
[6] Ho Tso-Hsui, Zh. Eksperim. i Teor. Fiz. **37**, 1825 (1959) [English transl.: Soviet Phys.—JETP **10**, 1288 (1960)]; D. R. Harrington, Phys. Rev. **120**, 1482 (1960).
[7] S. Weinberg, Phys. Rev. **112**, 1375 (1958).
[8] See, for example, L. B. Okun, Ann. Rev. Nucl. Sci. **9**, 82 (1959).
[9] T. D. Lee and C. N. Yang, Phys. Rev. **126**, 2239 (1962).
[10] Most of the data on hyperon leptonic decay are taken from the summary report of Rousset, Conference on Weak Interactions, Brookhaven National Laboratory, Upton, New York, September 1963 (unpublished), and C. T. Murphy, Phys. Rev. **134**, B188 (1964)].
[11] R. W. Birge, R. P. Ely, G. Gidal, G. E. Kalmus *et al.*, Phys. Rev. Letters **11**, 35 (1963).

leptonic decays of the Ω^- are observed. Still two other conditions are the $\Delta I=\frac{1}{2}$ rule for leptonic decays:

$$A(\Sigma^-\to n)=\sqrt{2}A(\Sigma^0\to p)\,, \tag{6e}$$

$$A(\Xi^0\to\Sigma^+)=\sqrt{2}A(\Xi^-\to\Sigma^0)\,. \tag{6f}$$

The first of these involves the unobservable Σ^0 decay and the other is not likely to be tested in the near future; evidence with respect to this $\Delta I=\frac{1}{2}$ rule from K-meson leptonic decays still seems to be ambiguous. Of the two remaining conditions one is the $\Delta I=1$ rule[12] for the unobservable decays among the Σ's:

$$A(\Sigma^-\to\Sigma^0)=-A(\Sigma^0\to\Sigma^+) \tag{6g}$$

whereas the other follows from Eq. (5).

$$A(\Sigma^+\to\Lambda)=\lambda A(\Sigma^-\to\Lambda)\,, \tag{6h}$$

where $\lambda=+1$ for first-class form factors and $\lambda=-1$ for second class. As emphasized by Weinberg[7] no firm evidence about the validity of Eq. (5) exists as yet.

3. CONSEQUENCES OF THE OCTET HYPOTHESIS

If we accept unitary symmetry for the strong interactions, it is useful to classify the currents $\mathcal{J}$ and $\mathcal{S}$ according to their transformation properties under SU_3. As far as hyperon leptonic decays are concerned, we may write with perfect generality if we accept the selection rules of Sec. 2:

$$\mathcal{J}=A_0\mathcal{J}(27)+B_0\mathcal{J}(10)+B_0'\mathcal{J}(\overline{10})+C_0\mathcal{J}(8d)+E_0\mathcal{J}(8f)\,, \tag{7a}$$

$$\mathcal{S}=A_1\mathcal{S}(27)+B_1\mathcal{S}(\overline{10})+C_1\mathcal{S}(8d)+E_1\mathcal{S}(8f)\,. \tag{7b}$$

Here $\mathcal{J}(N)$ and $\mathcal{S}(N)$ signify independent currents which transform under SU_3 like the $T=1$, $Y=0$, and $T=\frac{1}{2}$, $Y=1$ components, respectively, of the representation N.

We now define the most general form of the hypothesis that the weak-interaction current J transforms like components of an octet by the equation

$$A_0=A_1=B_0=B_0'=B_1=0\,. \tag{8}$$

These provide four additional relationships[13] to those discussed in Sec. 2. It must, of course, be emphasized that the octet hypothesis automatically gives the selection rules of Sec. 2 [except for Eq. (5)], a fact which is a major argument in favor of the hypothesis. Furthermore, the conserved vector current theory requires that at least the vector part of $\mathcal{J}$ transform like a component of an octet. Equation (8) allows for four independent amplitudes corresponding to C_0, E_0, C_1, and E_1. If we do not assume R invariance, which does not appear to be a good approximation, there is no invariant significance in assuming some ratio between C and E, since a bare interaction transforming like $8f$ will, in general, be renormalized by strong interactions so as to include $8d$, and vice versa. An exception is the conserved vector current, which is not renormalized and so remains as $8f$ at zero-momentum transfer. The present discussion is limited to general relations between amplitudes so that we shall not make use of this special feature of the vector current.

Of the four relationships following from Eq. (8), the two which have to do with the strangeness-conserving $\mathcal{J}$ involve the unobservable decays $\Xi^-\to\Xi^0+e^-+\nu$ and $\Sigma^-\to\Sigma^0+e^-+\nu$.[14] The other two may be written

$$A(\Xi^-\to\Lambda)=-\tfrac{1}{2}[(\tfrac{3}{2})^{1/2}A(\Sigma^-\to n)+A(\Lambda\to p)]\,, \tag{9a}$$

$$A(\Xi^-\to\Sigma^0)=\tfrac{1}{2}[(1/\sqrt{2})A(\Sigma^-\to n)-\sqrt{3}A(\Lambda\to p)]\,. \tag{9b}$$

By use of Eq. (6e) these may be written in a more symmetrical, although less useful form[15]

$$A(\Xi^-\to\Lambda)=-\tfrac{1}{2}[\sqrt{3}A(\Sigma^0\to p)+A(\Lambda\to p)]\,, \tag{9c}$$

$$A(\Xi^-\to\Sigma^0)=\tfrac{1}{2}[A(\Sigma^0\to p)-\sqrt{3}A(\Lambda\to p)]\,. \tag{9d}$$

Combining the two equations (9) we find

$$|A(\Xi^-\to\Lambda)|^2+|A(\Xi^-\to\Sigma^0)|^2=\tfrac{1}{2}|A(\Sigma^-\to n)|^2+|A(\Lambda\to p)|^2\,. \tag{10}$$

If we again make use of Eq. (6e) we see that this result is one we would expect if R invariance were valid, the sum over Λ and Σ^0 eliminates the R-noninvariant $d-f$ interference terms.

Considering only g_V and g_A couplings and neglecting the variation of the form factors with k^2, we may use Eq. (4b) to interpret Eq. (10) as a relationship between the transition probabilities for the four different leptonic decays involved. Since most of the available information concerns the Σ and Λ decays we shall try to substitute this information to find predictions about the Ξ leptonic decays. Present data[10] indicates[15a]

$$\begin{aligned}|A(\Lambda\to p)|^2&=(0.055\pm0.010)K\,,\\|A(\Sigma^-\to n)|^2&=(0.024\pm0.010)K\,.\end{aligned} \tag{11}$$

If we set[16] $\tau_{\Xi^-}=1.75\times10^{-10}$ sec, we obtain from Eqs.

[12] The most likely possibilities for studying the validity of this rule may be fast neutrino reactions producing π mesons.

[13] It follows from Eq. (5) that when first-class form factors are considered only the combination $10+\overline{10}$c ontributes while for second-class form factors it is $10-\overline{10}$. Thus setting $B_0=B_0'=0$ provides only *one* relationship.

[14] This statement is true for first-class form factors. There is, in general, an additional relationship given by Eq. (14), which is equivalent to Eq. (6h) for first-class form factors.

[15] In the octet hypothesis $\mathcal{S}$ transforms like a component of a spinor in U space and a vector in V space, using the notation of Levinson, Lipkin, and Meshkov, Phys. Letters **1**, 44 (1962); Nuovo Cimento **23**, 236 (1962); Phys. Rev. Letters **10**, 361 (1963). Equations (9) may be derived from the U-space equivalents of the $\Delta I=\frac{1}{2}$ rule Eq. (6e) and (6f) or the V-space equivalents of Eqs. (6g) and (14). Some observations as to the use of U and V selection rules for hyperon leptonic decays have been made by D. Horn (to be published).

[15a] *Note added in proof*. Equation (11) corresponds to a branching ratio $B(\Sigma^-\to n+e^-+\nu)$ of $1.3\pm0.5\times10^{-3}$ in excellent agreement with a recent result of $1.37\pm0.34\times10^{-3}$ [U. Nauenberg, P. Schmidt, J. Steinberger, S. Marateck *et al.*, Phys. Rev. Letters **12**, 679 (1964)] as well as earlier results (Ref. 10).

[16] H. Ticho, Conference on Weak Interactions, Brookhaven National Laboratory, Upton, New York, September 1963 (unpublished).

(4b), (9), (10), and (11) the following results[17] for the branching ratios B for Ξ^- decay.

$$B(\Xi^- \to \Lambda + e^- + \nu) \leq (1.0 \pm 0.2) \times 10^{-3}, \quad (12a)$$

$$(0.05 \pm 0.02) \times 10^{-3} \leq B(\Xi^- \to \Sigma^0 + e^- + \nu) \leq (0.16 \pm 0.03) \times 10^{-3}, \quad (12b)$$

$$B(\Xi^- \to \Lambda + e^- + \nu) + 9.2 B(\Xi^- \to \Sigma^0 + e^- + \nu) = (1.5 \pm 0.25) \times 10^{-3}. \quad (12c)$$

It follows from Eq. (6f), taking account of phase space and assuming $\tau_{\Xi^0} = 2\tau_{\Xi^-}$ (which follows from the $\Delta I = \frac{1}{2}$ nonleptonic rule), that

$$B(\Xi^0 \to \Sigma^+ + e^- + \nu) = 3.6 B(\Xi^- \to \Sigma^0 + e^- + \nu). \quad (12d)$$

Equation (12d) may be used to give alternative forms to Eqs. (12b) and (12c). Equation (12c) in either form represents the most definite prediction of the octet hypothesis. Since at the moment only Ξ^- leptonic decays have been studied and since the Σ^0 decay mode is probably indistinguishable from the Λ, we note the following alternative to Eqs. (12a) and (12b)

$$B(\Xi^- \to \Lambda + e^- + \nu) + B(\Xi^- \to \Sigma^0 + e^- + \nu) \leq (1.05 \pm 0.2) \times 10^{-3}. \quad (12e)$$

While early results seemed to give a larger branching ratio than this, it is too early to reach any conclusion.

4. SECOND-CLASS FORM FACTORS

The octet hypothesis does not necessarily yield the "standard behavior under G," Eq. (5). This may be seen by noting that a derivative-type coupling such as $(\bar{p}\sigma_{\mu\nu}\gamma_5 n)\partial_\nu$ may be introduced into J_μ in a manner consistent with the octet behavior.[18] Thus, combining the octet hypothesis with Eq. (5) yields additional results for strangeness-conserving processes.

To summarize the consequences of Eq. (5) we note that corresponding to the terms $\bar{p}n$, $\bar{\Xi}^0\Xi^-$, $\bar{\Lambda}\Sigma^-$, and $\bar{\Sigma}^0\Sigma^-$ in J_μ, there are in $GJ_\mu G^{-1}$ terms $\bar{p}n$, $\bar{\Xi}^0\Xi^-$, $\bar{\Sigma}^-\Lambda$, and $\bar{\Sigma}^-\Sigma^0$, respectively. Thus one can immediately obtain[7] from Eq. (5), the vanishing of second-class form factors from $A(n \to p)$ and $A(\Xi^- \to \Xi^0)$. In addition, one obtains Eq. (6h) and

$$A(\Sigma^- \to \Sigma^0) = -\lambda A(\Sigma^0 \to \Sigma^+). \quad (13)$$

Equation (13) coupled with Eq. (6g) yields the vanishing of f_3 and g_2 for $A(\Sigma^- \to \Sigma^0)$ and $A(\Sigma^0 \to \Sigma^+)$. Now the octet hypothesis provides the additional relationship

$$A(\Sigma^+ \to \Lambda) = A(\Sigma^- \to \Lambda), \quad (14)$$

which combined with Eq. (6h) yields the vanishing of f_3 and g_2 for $A(\Sigma^\pm \to \Lambda)$. From the conserved vector current hypothesis f_3 can be related[19] to f_2 and is found to be proportional to $(m_\Sigma - m_\Lambda)$, indicating that f_3 vanishes only in the limit when the mass difference between Σ and Λ goes to zero.

To extend this result to the strangeness-changing current $\mathcal{S}$ we introduce $G' = Ce^{i\pi V_y}$, where V_y is the y component of the V spin,[15] and postulate in analogy with Eq. (5):

$$G'\mathcal{S}^\mu G'^{-1} = -\xi \mathcal{S}^\mu. \quad (15)$$

Equation (15) follows from the octet hypothesis if $\mathcal{S}^\mu$ is written in a standard form bilinear in the baryon field with no derivative couplings. In the Cabibbo model (Sec. 5), in which $\mathcal{S}^\mu$ and $\mathcal{J}^\mu$ are components of the same octet, Eq. (15) follows as a consequence of Eq. (5); however, it appears reasonable to postulate Eq. (15) even though no useful relationships between $\mathcal{J}^\mu$ and $\mathcal{S}^\mu$ may exist. Corresponding to terms in $\mathcal{S}^\mu$ of the form $\bar{n}\Sigma^-$, $\bar{\Sigma}^-\Xi^0$, $\bar{\Lambda}\Xi^-$, and $\bar{\Sigma}^0\Xi^-$ there are in $G'\mathcal{S}^\mu G'^{-1}$ terms $\bar{n}\Sigma^-$, $\bar{\Sigma}^-\Xi^0$, $\bar{p}U_0{}^0$, and $\bar{p}U_1{}^0$, respectively, where $U_1{}^0 = \frac{1}{2}(\Sigma^0 - \sqrt{3}\Lambda)$ and $U_0{}^0 = \frac{1}{2}(\sqrt{3}\Sigma^0 + \Lambda)$. One can therefore obtain directly from Eq. (15) the vanishing of f_3 and g_2 for $A(\Sigma^- \to n)$ and $A(\Xi^0 \to \Sigma^+)$. In addition, one obtains

$$A(\Xi^- \to \Lambda) = -\lambda A(U_0{}^0 \to p),$$
$$A(\Xi^- \to \Sigma^0) = \lambda A(U_1{}^0 \to p),$$

which together with Eqs. (9c) and (9d) assures the vanishing of f_3 and g_2 for the remaining $\Delta S = 1$ amplitudes. Thus we reach the final conclusion that Eqs. (5) and (15) together with the octet hypothesis assure the vanishing of all second-class form factors.

This result is not very exciting, since when symmetry-breaking interactions are taken into account one can only conclude that f_3 and g_2 are small for strangeness-changing decays, a conclusion which might be expected in any case if f_3 and g_2 represent purely "induced" couplings. However, the result does serve to justify the neglect of the term linear in g_2 in Eq. (3) and the consequent use of Eqs. (4) in our discussion of decay rates.

5. CABIBBO MODEL

So far we have assumed that $\mathcal{J}$ and $\mathcal{S}$ are separately members of an octet but unrelated to each other. The only additional assumption which has in general an invariant significance within the framework of SU_3 symmetry is that $\mathcal{J}$ and $\mathcal{S}$ are members of the same octet. Specificially this means that in Eqs. (7) $\mathcal{J}(8d) + (E_0/C_0)\mathcal{J}(8f)$ and $\mathcal{S}(8d) + (E_1/C_1)\mathcal{S}(8f)$ are "π^+" and

[17] The inequalities result from considering A $(\Lambda \to p)$ and A $(\Sigma^- \to n)$ as two-dimensional vectors of known length but unknown relative angle. The errors are obtained by compounding the errors shown in Eq. (11). The error on A $(\Sigma^- \to n)$ is rather uncertain, particularly since this value was obtained by using data on $\Sigma^- - n + \mu^- + \nu$ as well as $\Sigma^- \to n + e^- + \nu$.

[18] It is interesting to note that for neutral nucleon-antinucleon states with $J = 1$ there are three possible combinations of C and P. See, for example, L. Wolfenstein and D. G. Ravenhall, Phys. Rev. **88**, 279 (1952). These are ${}^3S_1$ $(C = -1, P = -1)$, ${}^3P_1$ $(C = +1, P = +1)$, and ${}^1P_1$ $(C = -1, P = +1)$. These transformation properties are equivalent to those of the simple V, simple A, and the derivative-type axial coupling, respectively.

[19] J. Dreitlein and H. Primakoff, Phys. Rev. **125**, 1671 (1962).

"K^+" components, respectively, of a single octet. It is not assumed that $C_0=C_1$, which would also have invariant significance, since it is known that $\Delta S=1$ decays are weaker than $\Delta S=0$. What is assumed, however, is that

$$(C_1/C_0)=(E_1/E_0)=\tan\theta\,, \qquad (16)$$

where θ is the angle defined by Cabibbo,[3] and it is now required, that $\mathcal{J}(8d)$ and $\mathcal{S}(8d)$ are components of a single octet and similarly for $\mathcal{J}(8f)$ and $\mathcal{S}(8f)$. Equation (16) may also be written $(E_0/C_0)=(E_1/C_1)$, but, as noted before, it is not generally meaningful to specify a fixed value for this ratio. Equation (16) holds for both the V and A couplings, although for the generalized conserved vector current we would have the special case $C_0=C_1=0$. Equation (16) combined with the previous equation (8) yields a result essentially equivalent to the Cabibbo model. Other models, such as that of Cornwall and Singh,[4] either do not include Eq. (16), or, if they do include it, allow its consequences to be completely masked by symmetry-breaking effects.

In this model there are only two independent amplitudes, one for $8d$ and one for $8f$, instead of the four allowed in Sec. 3, and in addition there is an arbitrary parameter $\tan\theta$. Assuming perfect SU_3 symmetry, $\tan\theta$ is the same for all leptonic processes and so could be deduced (as done by Cabibbo) from mesonic decays. In this paper we consider only hyperon decays so that Eq. (16) simply provides one additional relation,[20] which relates $\Delta S=1$ decays to $\Delta S=0$ decays. The most useful form we can find for this is

$$\begin{aligned} A(\Sigma\to\Lambda)/[A(\Lambda\to p)-(\tfrac{3}{2})^{1/2}A(\Sigma^-\to n)] \\ =A(n\to p)/[(6)^{1/2}A(\Lambda\to p)-A(\Sigma^-\to n)] \\ =-\tfrac{1}{2}\cot\theta\,. \end{aligned} \qquad (17)$$

If we use the experimental results as given by Eq. (11) and assume as in Sec. 3 that the angle between the "vectors" $A(\Lambda\to p)$ and $A(\Sigma^-\to n)$ is completely unknown, we can deduce an upper limit on the rate $\Sigma\to\Lambda+e+\nu$, which is between one-quarter and one-half of the "UFI value." This limit depends essentially on the fact that $\Sigma^-\to n+e^-+\nu$ is reduced with respect to UFI at least as much as $\Lambda\to p+e^-+\nu$ and not on the particular values in Eq. (11). The experimental result that $\Sigma^-\to\Lambda+e^-+\nu$ appears in fact to be reduced below its "UFI value" represents a confirmation of the Cabibbo model, but not a very striking one. There are other models that also predict a reduced value.[19,21]

Equation (17) provides more specific predictions if it is combined with the conserved vector-current hypothesis (CVC). Since both sides of the equation are pure numbers the denominator of the left-hand side must be a pure A coupling in the limit $k^2\to 0$, where CVC allows no vector contribution[22] to $A(\Sigma\to\Lambda)$. When this information is combined with the numerical values of Eq. (11) and the fact that the denominator of the right-hand side must be a $V-1.2A$ coupling to agree with the numerator, two solutions for the angle between $A(\Lambda\to p)$ and $A(\Sigma^-\to n)$ are found. One of these corresponds to a reduction of the rate $\Sigma\to\Lambda+e+\nu$ to less than 5% of the UFI value, a result which seems to be ruled out by experiment. The other value for the angle yields a rate for $\Sigma\to\Lambda+e+\nu$ of 0.3 times the UFI value and a value of $\tan\theta$ in Eq. (17) equal to 0.25 in complete agreement with that deduced by Cabibbo from mesonic decays. When the errors in Eq. (11) are taken into account it is still found that the rate of $\Sigma\to\Lambda+e+\nu$ is less than 0.4 times the UFI value. If it is required that this rate be greater than 0.2 times the UFI value, then $\tan\theta$ is determined from hyperon leptonic decay to be within 20% of 0.25. It is this strikingly close agreement between the values of $\tan\theta$ deduced from different experiments that represents the major success of the Cabibbo model. The reason for this close agreement is hard to understand in view of the variety of values of $\tan\theta$ that can be deduced from different analyses of mesonic decays.[23] In this connection it is of didactic interest to try to apply Eq. (17) to the form factors g_3. To estimate these effective pseudoscalar form factors we use the pole term corresponding to a single π meson or K meson between the baryon and lepton vertices.[24] The first equality in Eq. (17) is then valid provided that the angle Θ that represents the d to f ratio of the *strong* pseudoscalar meson-baryon interaction is the same for K couplings as for π couplings. We find, however, that in place of $\frac{1}{2}\cot\theta$ in Eq. (17) we obtain

$$\tfrac{1}{2}\cot\theta(m_K/m_\pi)^2(g_\pi/g_K)\,,$$

where g_π and g_K are the strong coupling constants and $\tan\theta$ is deduced from the ratio of the rates for $K\to\mu+\nu$ and $\pi\to\mu+\nu$. In the perfect SU_3 limit this reduces to $\frac{1}{2}\cot\theta$; however, if as indicated by some experiments $(g_\pi/g_K)\simeq(m_K/m_\pi)$ we find the effective value of $\tan\theta$ for this case to be reduced by a factor of about 50. While this is certainly an extreme example it serves to illustrate the large effects that the symmetry-breaking strong interactions may have when attempts are made to compare $\Delta S=0$ and $\Delta S=1$ weak interactions.

With the aid of CVC this model also gives definite predictions for the values of the "weak magnetism" form factors f_2. The derivation is exactly as for the case

[20] In contrast to the relations discussed in Secs. 2 and 3, Eq. (17) is not linear in the amplitudes. As in the other cases, this relation applies separately for each form factor or for the amplitudes considered as vectors. It is not necessary to define the ratio of two vectors since the equation requires that numerator and denominator be "parallel" vectors. Because of this requirement it might be argued that Eq. (17) really involves more than a single relationship.

[21] Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) 30, 836 (1963).

[22] The vector contribution for $k^2\neq 0$ may be calculated (Ref. 19) from CVC [cf. Eq. (18a)]. Assuming the decay $\Sigma\to\Lambda+e+\nu$ is not too inhibited the main contribution to A $(\Sigma\to\Lambda)$ will still come from the axial-vector contribution at the physical value of k^2.

[23] M. Gell-Mann, Phys. Rev. 125, 1067 (1962); J. J. Sakurai, Phys. Rev. Letters 12, 79 (1964); see also Ref. 4.

[24] See Ref. 5; also L. Wolfenstein, Nuovo Cimento 8, 882 (1958); E. M. Ferreira, Nuovo Cimento 8, 359 (1958).

of the magnetic moments[25] and yields among the results

$$f_2(\Sigma^- \to \Lambda) = -(m_\Sigma/2m_N)(\tfrac{3}{2})^{1/2}\mu_n = 1.5\,, \quad (18a)$$

$$f_2(\Sigma^- \to n) = -(m_\Sigma/2m_N)(\mu_P + 2\mu_n)\tan\theta = 0.32\,, \quad (18b)$$

$$f_2(\Lambda \to p) = -(m_\Lambda/2m_N)(\tfrac{3}{2})^{1/2}\mu_P \tan\theta = -0.33\,, \quad (18c)$$

where μ_P and μ_n are the anomalous nucleon magnetic moments in units of $(1/2m_N)$. The main effect of f_2 shows up in the electron spectrum[6] (and other detailed observables) which contains a term proportional to $4Rg_1 f_2$ rather than in decay rates which involve f_2 only in terms proportional to R^2. For example, the values given yield a distortion of the spectrum for $\Lambda \to p + e^- + \nu$ decreasing the number of very low-energy electrons and increasing the number at the top of the spectrum by about 20%.

6. DISCUSSION

We inquire now as to what may be concluded from possible experimental failures of the relationships discussed. Failure of the rules based solely on strangeness and isotropic spin (Sec. 2) would mean that the basic weak-interaction current is not a member of an octet, since, starting with an octet, the (nonelectromagnetic) symmetry-breaking interactions could at most mix in the same isospin and hypercharge reproduce ing the general form Eq. (7). Thus, if those experiments on K^0 decay that indicated $\Delta Q = -\Delta S$ should be confirmed or the comparison of $K^+ \to \pi + e + \nu$ to $K^0 \to \pi + e + \nu$ should definitely indicate the failure of the $\Delta I = \frac{1}{2}$ rule, then the octet hypothesis for the weak current must be abandoned.

One model we have considered that allows $\Delta Q = -\Delta S$ decays consists of two weak-interaction currents each coupled to itself but not to the other.[26] Besides the leptonic current, one contains a $\Delta S = 0$, $\Delta I = 1$ current with a small admixture of $\Delta S = -1$, $\Delta Q = -\Delta S$, $\Delta I = \frac{3}{2}$ current whereas the other contains a $\Delta S = 1$, $\Delta I = \frac{1}{2}$ current with an admixture of $\Delta S = +2$, $\Delta I = 0$. These four currents have the charge and hypercharge characteristics of the four positively-charged members of a decuplet. Thus, a possible extension of this theory would include the assumption that $\mathcal{J}$ and $\mathcal{S}$ discussed above were members of a decuplet so that $A_i = C_i = E_i = 0$ in Eq. (7). This leads to more specific predictions than the octet hypothesis. It gives the result $|A(\Lambda \to p)|^2 = 1.5|A(\Sigma^- \to n)|^2$, a result which may just barely be compatible with experiment [Eq. (11)]. It also gives the result $|A(\Xi^- \to \Lambda)|^2 = |A(\Lambda \to p)|^2$ thus predicting the branching ratio for $\Xi^- \to \Lambda + e^- + \nu$ slightly larger than the upper limit given by the octet assumption, Eq. (12a). Of course, a major argument against the decuplet assumption is that the vector part of $\mathcal{J}$ is no longer given by the conserved vector current and that the axial part no longer allows $\pi \to \mu + \nu$.

If the rules of Sec. 2 are found to be valid, but the further consequences of the octet hypothesis [Eqs. (9)] are violated by the leptonic decays it is necessary to decide whether the violation is due to the approximate character of SU_3 for the strong interactions or to the failure of the hypothesis. There are qualitative reasons for believing that the SU_3-violating mass differences would have a relatively small effect in causing violations of Eq. (9) because of the symmetrical manner in which the masses enter and because the same intermediate states are involved in the dispersion relation analysis of each of the vertices entering the equation. Thus, while corrections of the order $(m_\Lambda - m_N)/m_N$ can naturally be expected, a sizable violation of these equations must raise serious doubts about the octet hypothesis.[27]

In using Eqs. (12) to test the validity of Eqs. (9) approximations have been made whose validity must be considered. For the values of k^2 involved the neglect of the variation of the form factors with k^2 should make less than a 10% error; furthermore, the effects of this variation would partially cancel out in the comparison of the Ξ decays with those of Σ and Λ. Reasons for neglecting the contribution of form factors other than f_1 and g_1 have been given in Sec. 4.

The Cabibbo model contains additional assumptions about the weak-interaction current beyond those of the most general octet hypothesis, but there are reasons (exemplified by the extreme example in Sec. 5) for believing that the consequences of these assumptions may be masked by the symmetry-breaking strong interactions. Thus, it would not be easy to draw definite conclusions about the weak-interaction current from the failure of some of the predictions that follow from Eq. (17).

We therefore conclude that the most clear-cut tests of the octet hypothesis are those presented in Sec. 3, assuming that the isospin and strangeness selection rules of Sec. 2 are valid. Consequently, it is very important to obtain additional information on Ξ leptonic decays.

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[25] S. Coleman and S. Glashow, Phys. Rev. Letters **6**, 423 (1961). The absolute signs in our Eqs. (18) depend on our sign conventions, which are those of Ref. 1. To use Eqs. (18) it is necessary to have g_1 and f_1, which with our sign convention and our solution for the Cabibbo model are $g_1(\Lambda \to p) = -0.20$, $f_1(\Lambda \to p) = -0.30$, $g_1(\Sigma^- \to n) = +0.10$, $f_1(\Sigma^- \to n) = -0.25$, $g_1(\Sigma^- \to \Lambda) = +0.65$. It may also be noted that Eq. (18a) follows from the more general hypothesis of Sec. 3 together with CVC. Results for f_2 have also been given by N. Cabibbo (unpublished).

[26] L. Wolfenstein, Nuovo Cimento **29**, 859 (1963).

[27] It may also be noted that Eq. (10), which is a combination of the two equations (9), also follows from a pure decuplet hypothesis or a pure 27 hypothesis together with the $\Delta I = \frac{1}{2}$ rule for the current $\mathcal{S}$. Thus the violation of Eq. (10) would indicate an admixture of at least two of the three representations 8, 10, and 27.